Lightweight Mirror Systems for Spacecraft An Overview of Materials & Manufacturing Needs

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Abstract—The purpose of this paper is to provide an introductory overview of large space mirrors, conventional mirror materials, and manufacturing methods, and suggest possible materials, processes, and manufacturing research and development approaches to enable the large lightweight mirrors required for future space systems.

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1. BACKGROUND

The Department of Defense (DoD), National Aeronautics and Space Administration (NASA), and other U.S. government agencies have a number of space-based optical systems on the drawing boards, ranging from 8-100 meter diameter apertures. These planned and proposed programs such as the Space Based Laser (SBL), the Next Generation Space Telescope (NGST), and other remote sensing programs have identified a need for large, lightweight, low cost, simple, reliable, and producible designs for large-scale Performance of these systems deployable optics. dramatically improves as mirror size (aperture) increases. However, for a given equivalent thickness, mirror mass increases by the square of the mirror diameter and optical system mass depends strongly on the mass of the primary mirror. Lightweight mirrors are needed because they lead to large savings in payload mass, which in turn leads to tremendous savings in launch cost. The time and cost for manufacture of spacecraft mirrors has been the prime schedule and cost driver for past large space telescopes/optics. For NGST, systems studies have shown that the primary mirror is the chief driver of system mass. For future high energy laser systems, the mirror will be even larger and therefore an even larger driver of system weight.

Due to launch vehicle size and volume constraints, structures larger than four to five meters in diameter must be deployed in space. Deployable mirrors are complicated systems comprised of multiple elements. It is necessary that the segments be adjustable to ensure the correct shape (figure) of the primary mirror in its deployed state and provide a method for rapid control of the mirror figure to maintain responsiveness during changing operational conditions. Deployment of a system in space has yet to be demonstrated to the degree of accuracy required for a telescope operating in the visible or near infrared (IR) wavelengths.

Telescope Mirrors

Reflecting telescopes use the focusing property of paraboloidal mirrors to concentrate and magnify light. Paraboloidal mirrors can perfectly focus light from distant objects into an image near the mirror. The F/number, or focal ratio, of the mirror is the ratio of the focal distance to the diameter and indicates how many mirror diameters above the mirror surface the image is formed. The primary mirror consists of a paraboloidal surface coated to reflect light at the wavelength of interest. In a typical configuration, the light is reflected from the primary mirror and focused onto a secondary mirror, which directs the light to a detector. Figure 1 shows 3 types of basic telescope designs utilizing this type of mirror. For laser projection applications, the light path is reversed and the telescope is referred to as a beam director. The beam director directs and focuses laser light at a far distance (hundreds to thousands of kilometers).

The surface of a telescope mirror must have a precise geometrical figure typically within approximately 1/10 to 1/20 root mean square (RMS) of the wavelength of light. Since small-scale roughness will cause light to be scattered, inaccuracies on smaller scales, typically referred to as finish or micro-roughness, must be controlled down to about 1/200 to 1/500 of a wave RMS. This imposes severe constraints

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on the manufacture, test, and maintenance of optical figure and finish.

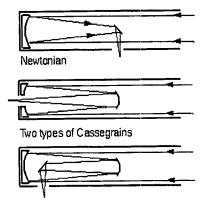


Figure 1. Reflecting Telescopes [10]

Large Terrestrial Mirrors

Reflecting telescopes for terrestrial observatories have traditionally been built around simple monolithic glass primary mirrors. Low expansion glass was chosen because of its relative ease in polishing to the required surface roughness tolerances (< 2 nanometers root mean square surface roughness (nm RMS)) and its low coefficient of thermal expansion near room temperature. Because glass is a low specific stiffness material (see Table 1), a classical thickness/diameter ratio of 1:6 was developed over the years to ensure dimensional stability. If you scale this technology to a diameter of 6.5m, a solid blank will weigh about 60 tons. When produced in sizes larger than 5 to 6 meters in diameter, monolithic mirrors are extremely heavy and expensive, difficult to manufacture, difficult to handle, difficult to support, susceptible to thermal shock, and inappropriate for space applications.

Traditional Steps for Producing a Monolithic Glass Mirror:

- 1. Start with a flat (plano) or spin cast (concave paraboloid) glass blank.
- 3. Polish the surface finish to an accuracy of better than 25 nm RMS with a lap using a very fine polishing compound. (The surface figure and finish can be directly measured during these processes with a direct contact profilometer.) Additional, post finishing, noncontact figuring may be required to achieve the final figure required by the mirror. This can be accomplished by ion figuring using a directed inert neutralized ion beam, plasma assisted chemical etching (PACE), and argon or oxygen ion beam milling.
- Finally, the substrate is turned into a mirror by applying a very thin metal or dielectric coating. The coatings are

typically applied in a vacuum chamber by depositing a small amount of metal onto the clean mirror surface.

Lightweighting of Mirrors

Because of the difficulty in supporting and handling heavy mirrors, large terrestrial mirrors are commonly "lightweighted" by a factor of five or more. Since only the topmost reflecting layer is required for a mirror to function, a mirror can be lightweighted by removing material from the rear by machining (computer controlled diamond grinding, abrasive water jet, laser cutting) and/or etching to produce a web stiffened mirror. Alternate methods to produce web stiffened mirrors include fusing or bonding a thin facesheet, core, and back-plate together to form a lightweight mirror and casting the mirror with the web stiffeners in place. An 8.4m, f/1.14 lightweighted borosilicate glass mirror that was produced by the University of Arizona for the Large Binocular Telescope, Mount Graham, Arizona is shown in figure 2. This mirror was cast in place around removable lightweighting mandrels in a spinning furnace. Another alternate method is to support a very thin meniscus sheet on an actuator array attached to a lightweight reaction structure. An example of a thin meniscus mirror is shown in figure 3. The University of Arizona prepared this mirror for NASA's Next Generation Space Telescope Mirror System Demonstrator (NMSD). The mirror substrate was prepared by machining the substrate thickness until only a very thin meniscus remained (~2 mm).

The webs in a web stiffened design and the actuators in a meniscus mirror design provide stiffening and support for the thin facesheet. Since the facesheet is thin, minute deformations (quilting and web print through) between webs can occur during polishing. Sagging of the facesheet between supports due to gravity can also occur. It is important to support the mirror during machining, polishing, and operation. Also, since glass is a brittle material, as the glass is made very thin, greater care must be taken in handling and supporting the mirror to prevent breakage. Finally, since glass is a low stiffness material, lightweighting leads to a low first mode frequency. This causes the mirror to deform in the presence of low frequency vibrations which leads to image distortion.

Both of these methods for producing lightweight mirrors are labor and machining intensive and the mirrors are vulnerable to breakage throughout manufacture, handling, and shipping. Also, the state of the art in weight savings by these techniques does not allow production of stiff mirrors with extremely low areal densities.

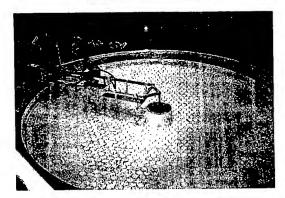


Figure 2. 8.4m, f/1.14 lightweighted glass mirror for the Large Binocular Telescope, Mount Graham, Arizona [12]

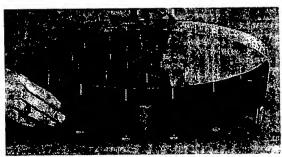


Figure 3. Thin meniscus glass with reaction structure and actuators (University of Arizona) [11]

Space Mirrors

The practice of using glass mirrors continued into satellite systems. The Hubble Space Telescope primary mirror measures 2.4 m (8 ft) in diameter and weighs about 826 kg (1820 lbs.). It is constructed of honeycomb lightweighted Corning ULE (ultra-low expansion) silica/titania glass and coated with a thin layer of protected (overcoated) aluminum to reflect visible light.

However, proposed space systems require larger mirrors than can currently be build as a monolithic mirror, and

which due to limited size of launch vehicles, drive a fundamental design change – deployable mirrors. One approach for deployable mirrors is to divide a primary mirror up into smaller, launchable segments and reassemble them on orbit into a full size mirror. One such approach, a mirror built from hexagonal segments, is illustrated in figure 4. Other segment shapes such as pie, "keystone", arc, and "drop leave" shaped segments have been investigated and have their advantages and disadvantages.

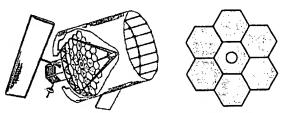


Figure 4. Large Segmented Mirror Space Telescope Concept

Another major driver for large space mirrors is cost. Since the cost goes up for mirrors by approximately the square of the diameter, large mirrors are cost prohibitive. However, since the majority of the cost for conventional mirrors is in the manufacturing (grinding, polishing, etc.) to get to the optical tolerances required, lower cost processes would result in significant cost savings. Therefore, there is a large payoff for large space mirrors if the manufacturability of mirrors can be improved.

For an assembled, segmented mirror, each segment must be properly aligned with the others to effectively produce one large mirror. This is performed with a figure control system as shown in figure 5 and comprised of four major parts: segmented mirror and its support, position and/or force actuators, figure sensor, and control hardware and software.

Position or force actuators in the figure control system allow the position of each segment to be controlled in relation to each other. Additional actuators may be required to apply bending forces to the mirror to adjust the figure of the segment. This allows for correction of on-orbit errors. In order to accomplish this, additional control is required for the increased number of actuators. Examples of actuators are shown in figure 6.

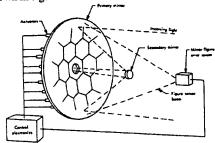


Figure 5. Segmented Mirror with Figure Control

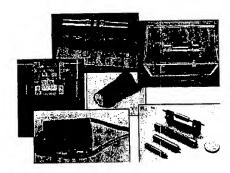


Figure 6. Examples of Mirror Actuators [11]

Membrane Mirrors

An innovative concept that is being considered for large lightweight space mirrors is membrane mirrors. This concept involves the utilization of very thin films or foils as the mirror substrate. The concept grew out of a concept for large inflatable antennas, in which a paraboloidal radio frequency reflector (rf) is the inside back layer of a large transparent balloon supported by an inflated torus (Figure 7.). Use of this concept for large mirrors introduces some major issues. The paraboloidal figure and surface finish of an optical mirror must be several orders of magnitude better than for an rf antenna and the light absorptance, scatter, and refraction of the transparent front polymer layer forming the balloon would prevent adequate performance of the mirror.

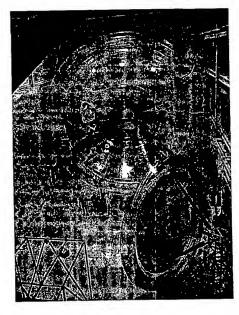


Figure 7. 5m and 10m Diameter Inflated Paraboloidal SRS Technology Inc. Reflectors at AFRL/VS, Kirtland AFB, NM [13]

Methods are being investigated at the Air Force Research Laboratory (AFRL/VS, DE, and ML) to determine if a self supporting, or inflated structure supported paraboloidal membrane mirror can be formed from existing polymeric materials, and if these mirrors can be used along with optical correction techniques as a primary optical mirror. It is clear from the initial work that the concept is feasible, but conventional materials and processes are insufficient to produce a mirror at this time. Materials development must be performed in several areas to produce this type of mirror.

A membrane mirror substrate material must be able to be formed into a thin, paraboloidal shape and rolled into a small deployable package. Because of this requirement, polymeric materials have been investigated for this purpose. Other materials such as thin metal films may be considered if sufficient flexibility can be demonstrated. Various methods have been considered to form the polymer into a mirror quality paraboloidal shape. These include forming the film by spin casting on a solid or liquid mandrel, and casting the polymer film on a mirror quality mandrel. An advantage of spin casting on a liquid mandrel is the inherent scalability of the process, but the choice of materials that can be cast upon a liquid are limited. Spin casting on a solid concave mandrel increases the number of potential materials to form the membrane but a large solid mandrel must be produced. An advantage of this method is that the mandrel does not need to be of optical quality to form an optical quality membrane surface. The back side of the mirror will take on the shape of the mandrel while the front of the membrane will form the paraboloidal figure dictated by the spin rate. Additional information on spin casting is included below in the glass section. Casting or depositing the film on a non spinning mandrel allows for the choice of the greatest number of materials since the method does not rely on the flowability of the material. Materials could also be applied by deposition techniques such as vapor deposition. Since the mandrel does not spin, this approach requires the use of an optical quality convex mandrel to allow direct replication of the mandrel surface onto the membrane surface (as discussed in the replication process section).

A membrane mirror could be produced by these methods whole or in smaller pie shaped segments and joined together following subsequent coating processes. A membrane mirror will require more than just a reflective coating to perform its function. Coatings will be required to allow the adjustment of the mirror shape just as actuators are needed for segmented mirrors. Another concept is to utilize a shape adaptive coating such as a shape memory alloy coating to perform the actual deployment of the film, thus eliminating the need for an inflatable structure altogether. Research is required to develop shape adjustable coatings or membranes for these purposes. Further information on these coatings is included below in the actuator materials section. Other concerns for membrane mirror materials include puncture and tear resistance, lack of wrinkling or coating damage

during stowage, self healing, and atomic oxygen resistance or protection.

Although useful for conventional size mirrors, if membrane mirrors are proven as a practical technology, they could also be scaled to very large sizes (30-100 m and greater) than would be unattainable by conventional segmented mirrors.

2. MATERIALS & MANUFACTURING

To minimize spacecraft weight, a stiff (high modulus) and lightweight (low density) structure is required to provide structural integrity during launch and operations. Also, mirror materials must possess a low coefficient of thermal expansion (CTE) and high CTE uniformity to ensure dimensional stability over thermal swings during normal use. Use of a higher CTE material greatly reduces the allowable use temperature envelope and increases the need for active thermal control and high level actuation to minimize and correct distortions in mirror shape. Also, in order to maintain dimensional stability, mirror materials must be isotropic. Finally, mirror materials must be able to be produced into mirror systems. This means the material needs to be producible to the required scale; the material must be able to be machined and polished to optical tolerances as required (<2.5 nm RMS); and the material must be coatable with a proper optical coating to reflect the optical wavelengths required for its application.

Mirror Substrate Materials:

Large mirror systems are expensive and time consuming to produce. Because of this, the mirror community is very conservative in its selection of materials and use of new materials. As a result, very few materials are used for large mirror applications and new materials are considered unacceptably high risk until demonstrated at a larger scale than that common to most structural applications. To be accepted for a mirror demonstration of a 1.5-3m segment, a new material needs to be scaled and demonstrated to at least 0.5m and available in 1.5-3m sizes for user confidence in the technology.

Table 1 lists a selection of materials and properties of some common traditional and non-traditional mirror substrate materials. The table is color coded to show which properties are extremely desireable (blue), desirable (green), acceptable (yellow), and undesirable (red) for mirrors. The more green that is showing for a particular material, the more desireable a material is for use as a mirror substrate. For Black and White copies of this paper, a color code is included at the bottom of the table.

The following is a discussion of the merits and disadvantages of various mirror materials and processes grouped into classes of materials. For each material class,

an overview of applicable processes and potential future research and development are discussed.

Glass / Glass Ceramics:

Glass is the historical and current material of choice for large mirrors operating near room temperature. Glass can be polished to a very smooth and precise surface without any grain structure. Several glasses have been tailored to have a coefficient of thermal expansion close to zero to allow production of thermally stable optical components. These glasses include silica based glasses such as ULE (ultra-low expansion) glass from Corning and Zerodur glass from Schott. Other glasses are less expensive and easier to procure but do not exhibit as low a CTE and therefore are less thermally stable and used in less demanding applications such as small telescope mirrors. These include borosilicate glasses such as Ohara's E6, Schott's BK7, and Corning's Pyrex. Regardless of the glass used for mirrors, glass has an inherently low stiffness that leads to poor dynamic performance and therefore becomes a less desireable material as mirrors are made larger and lightweighted. Other drawbacks of glass include its high cost, poor handlability, and long lead time for the procurement of large glass blanks.

Because of the process control needed to produce near zero CTE glass, Zerodur and ULE are only available as large blanks from the manufacturers following a very long procurement lead time. After receiving the large glass blank needed to produce a mirror, machining and polishing is required. This is difficult, requires a long time, and is very expensive (~60% of the cost of a mirror). Alternate manufacturing methods are needed to eliminate the time and cost of glass mirror production.

Borosilicate glasses can be supplied in bulk form and melted into a usable mirror blank form. This greatly reduces mirror lead time at a sacrifice of dimensional stability due to borosilicate glass's higher CTE. One method for producing mirror blanks from borosilicate glass is by spin casting. Spin casting allows the production of a rough paraboloidal shape by casting glass in a spinning furnace. Spin casting at the proper speed while the glass is molten allows centrifugal forces to shape a natural paraboloidal curve in the surface of When cooled, the mirror surface is the molten glass. accurate to a small fraction of an inch - close to the mirror's final paraboloidal figure. Rotation speed of the furnace is determined by the desired mirror focal length according to the formula: Focal Length, $L = (g / 2\omega^2)$, where g is the force of gravity and ω is the rotational speed of the furnace. This method greatly reduces the amount of raw material and machining needed to create the shape from a flat blank. If cast as a flat and ground to a paraboloid, a 6.5m mirror would waste 12 tons of glass and one year of extra work. Spin casting of glass does not produce an optical figure or finish, therefore, a mirror blank produced by spin casting

Table 1. Materials Properties

	Dag auticas	Density	CTE @ RT	CTE @ Cryo	Thermal Conductivity (at RT (k)	Thermal Conductivity at Cryo (k)	Specific Heat (c)	Young's Modulus (E)	Specific Stiffness (E/p)	Polishing rate
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still needs to be machined and polished to produce an optical tolerance surface. It may be possible to optimize the spin casting process to eliminate the need for subsequent machining and therefore reduce the cost of manufacturing glass mirrors. Possible improvements for spin casting of glasses includes spin casting of sol-gel derived glasses rather than melted glass. This would allow low temperature spin casting which may improve the net shape surface roughness and reduce the cost of the raw materials. Control of shrinkage and porosity of sol-gel glass would need to be improved. Use of a sol-gel approach would also allow the easy incorporation of dopants, additives, fillers, or fibers to allow tailoring of stiffness and CTE.

Another method that shows promise to simplify manufacture of mirror segments and greatly reduce cost is mirror replication. Instead of investing significant time and money on machining of each mirror segment, in replication, the machining time is spent producing a polished convex master from which mirror replicas are produced. A perfect replication process would yield a mirror segment requiring no additional machining or polishing.

A sample replication process is described in the following and is illustrated in figure 8:

- a) Grind & polish the inverse shape of a mirror into a master mold. The master must be stable at the temperatures required to process the mirror material. The master and/or mirror materials may change shape change during the process due to CTE differences or shrinkage during cure. This must be taken into account in the design of the master so that the final mirror has the correct optical figure and finish
- b) Vacuum deposit thin release layer onto mold. Potential release agents depend on the mold and mirror materials and include vacuum deposited gold, silver, silicone oils, Teflon, sodium fluoride, and others.
- c) Slump glass to the replication master and cool. Any trapped air will prevent replication to optical tolerances. Depending on material an upper mold may be needed for application of pressure.
- d) Release the mirror from mold and trim edges.

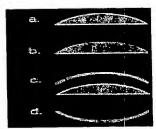


Figure 8. Schematic of Replication Process [7]

Because of the CTE mismatch between the replication master and the glass and the need to remove all air from between them, this process does not work well for large size

mirrors and additional machining of the mirror is still required. However alternatives to the process may allow this process to produce low cost mirrors. Possible alternatives include producing the master from the same material as the mirror to eliminate CTE mismatches, melting glasses onto the mold (rather than slumping) to prevent trapped air, applying glass as a thick coating by vacuum deposition (ULE is currently manufactured by vacuum deposition of bolus), applying glasses from powder form by plasma spray, or applying glass to the mold in a low temperature sol-gel process.

Additionally, the replication process may be used for other material classes such as thermoplastic polymers, UV or low temperature curable thermoset polymers, ceramics, and composites. Issues of the replication process with these materials will be discussed separately below.

These methods for producing net shape glass mirrors may also be adapted to produce glass matrix composite materials. The addition of carbon or ceramic fibers to glass would yield a glass composite with a much higher stiffness and improved handlability. Additional information on composites is discussed below.

Since machining of glass is a large percentage of the cost of glass mirrors, low cost alternatives to mechanical machining are required. Machining is expensive due to the time, cost of specialized equipment, and extreme amount of monitoring and control required. Improved methods for quickly measuring the surface of the mirror during machining are required as well as methods for quickly and accurately removing excess material from the surface. Currently ion beams and plasmas are used to remove small amounts of material from the mirror surface for final optical figuring. Ideally, similar processes could be used to provide all material removal, allowing quick, accurate, touch free machining and polishing. Possible processes include evaporation or sputtering with directed ion beams, pulsed lasers, and microwave sources. These processes could be applied to the materials classes below as well.

Finally, another potential process for glass to replace machining is material addition. Instead of removing material to achieve the correct mirror shape, it may be possible to add material where needed to produce the correct shape. Possible processes to accomplish this include deposition and sol-gel processes. In order to accomplish this, large scale deposition techniques would be required and the process would require a high degree of process monitoring and control to ensure application of the correct amounts of material in the correct locations. These processes could be applied to the materials classes below as well.

Metal:

Metals have been used as mirrors for many terrestrial applications due to their machinability, handlability, and cost. Aluminum has been used extensively due to its ease of machining and polishing and because it is highly reflective and does not need to be coated to yield high broadband reflectivity. Aluminum has a higher specific stiffness than glass, which makes it a potential low cost alternative. However, aluminum has a high CTE, which limits its use to small optics. As an alternative to aluminum, beryllium has been used extensively as a mirror for cryogenic applications. Beryllium has a low density coupled with a high stiffness, which makes it very attractive for lightweight applications. The reason beryllium is used for cryogenic applications is it possesses a near zero CTE at cryogenic temperatures (below 70K). However, beryllium's CTE at room temperature, although lower than Al is still high and therefore limits its usefulness for room temperature applications. Other issues with beryllium include high cost, toxicity, slow machinability, and anisotropic behavior which limits the methods (hot isotactic pressing or vacuum hot pressing of small randomly oriented powders) by which beryllium can be prepared.

Creating a composite of aluminum or beryllium and carbon fibers would allow CTE to be tailored while retaining low density and high stiffness. Since beryllium would not require the carbon fiber to be continuous to provide stiffness to the composite, very short, small diameter carbon fibers could be employed for CTE matching. These could be directly mixed with the beryllium powder prior to pressing. Fiber interface materials would need to be investigated to prevent the formation of beryllium carbide.

It may be possible to employ plasma spray techniques (safety concerns may exist) with aluminum or beryllium powders to produce large mirrors without the need for large vacuum ovens or presses. This would reduce the cost of the metal blanks and may allow the production of replicated optical surfaces.

Ceramic:

Recently, small mirror systems have been produced in silicon carbide. Silicon carbide has a very high stiffness and an adequate combination of the critical mirror properties. Also, it has promise for uses at both room temperature and cryogenic applications. Different forms of silicon carbide are available and properties and manufacturability vary. Reaction bonded silicon carbide is a two-phase material consisting of two interpenetrating networks of alpha silicon carbide and silicon. It is difficult to polish to low values of surface figure and roughness. Chemical vapor deposited (CVD) Beta- silicon carbide is theoretically dense, which overcomes issues of porosity and multiple phases and can be polished to a surface finish of <1 nm RMS. However, it is

produced by an expensive multi-step fabrication process and scale up issues exist. Hot pressed silicon carbide is not theoretically dense and its porosity leads to poor surface roughness and requires substantial machining in order to be lightweighted. Finally, melt infiltrated silicon carbide may allow near or net shape mirrors to be formed but the approach has not been proven.

Some ceramics that may also be useful as mirror materials include boron nitride due to its low density and low CTE, blackglass (SiO_xC_y), and high modulus alumina fiber reinforced ceramic composites.

Alternate processes for production of ceramic structures should be investigated. These include sol-gel processes and plasma spraying. Both sol-gel and plasma spray techniques allow mixing of multiple types of materials into a single bulk material. Therefore, they are both useful for tailoring properties such as stiffness and CTE.

Many ceramics can be processed by plasma spray techniques. Plasma spraying is a low cost approach to produce solids from powders without the use of a large press or furnace. Bulk ceramic and ceramic matrix composites could be produced by plasma spraying onto a replication mold to net or near net shape and machined as needed.

Carbon:

Carbon in its many forms has a low density and low CTE and in most forms is easy to machine. In fiber form, it has an extremely high stiffness which, coupled with its low density, is why it is used extensively in the production of high stiffness or high strength lightweight composites. Also in fiber form, it has a negative CTE and when combined with other materials in a composite can provide tailored zero CTE composites. Carbon has the potential to be an excellent mirror material either as a bulk material, a composite material (carbon-carbon, ceramic matrix, or polymer matrix), or as a lightweighted foam.

In order for carbon to be used in a low earth space environment, carbon must be coated with a protective coating to prevent erosion by atomic oxygen. In most cases the mirror's optical coating can provide this protection. In order to prevent a bimetallic effect due to the coating and to protect the back surface of the carbon, both sides of the structure need to be coated.

Polymer:

Polymers are used extensively in small optical applications such as eyeglass lenses, lenses for compact disc players and paraboloidal mirrors for flashlights. They offer an opportunity to mass produce by replication low cost lightweight parts with very little or no need for machining. However, inherent limitations to the use of polymers is their

very high CTEs, their absorption of water which leads to a coefficient of moisture expansion (CME) which must be taken into account when used in a vacuum, and their normally low stiffness compared to most other materials. The chief advantages of polymers include: low cost, light weight, low to no heat needed for processing, ease of modeling, and high shock resistance. A potential advantage of polymers is the ability to tailor adaptive nature into polymers such as shape change, which may allow production of adaptive mirrors with out the use of heavy, bulky mechanical actuators.

The primary disadvantages of polymers are their high CTE and low stiffness. However, highly aligned crystalline polymers such as aramid (Kevlar) fibers are noted for their negative CTE and high stiffness. Currently, aramid fibers are combined with higher CTE polymers to produce mixtures with tailored CTE (7-10 ppm/°C) for electronic packaging. Increasing the volume fraction of aramid fiber would allow production of tailored zero CTE polymers and increase the stiffness. Also, because of the tailorability of polymers, it may be possible to produce polymers with highly crystalline chain segments coupled with amorphous/glassy segments that yield single polymers with low or zero CTE.

Polymers hold the promise for producing net shape mirrors by various methods such as spin casting and replication. In addition, alternate forms such as composites, nanocomposites, and foams may yield high stiffness alternatives to glass. Because of their low stiffness, polymers are commonly used with carbon fibers to form high strength or stiffness composites. These are described separately below.

In order for most polymers to be used in a low earth space environment, the polymers must be coated with a protective coating to prevent erosion by atomic oxygen. In most cases the mirror's optical coating can provide this protection. In order to prevent a bimetallic effect due to the coating and protect the back surface of the polymer, both sides of the structure need to be coated.

Composite:

Composites have found use in most applications requiring low mass and high stiffness. Composites allow the mixing of different materials (fibers and matrices) to produce a tailored material with optimized properties. In order to optimize stiffness in composites, carbon (or graphite) fibers are used as a very stiff, reinforcing material. Since carbon fibers have a negative CTE, selection of matrix materials (type and percentage) allows tailoring of CTEs to zero. Many different matrix materials are available and selection depends on the application. The most common composites for space applications involve polymeric matrices such as epoxy, cyanate ester, and siloxane materials. Because they

are produced from polymeric materials, the resultant composites have low density, high stiffness, and can be produced at low temperatures, limiting thermal distortions. Metals, ceramic (SiC), and glass materials can also serve as matrix materials for composites. Also, other fibers such as glass, boron, aramid, polyethylene, and ceramic are often used for tailored properties. A benefit of this type of composite for mirror applications lies in the ability to tailor CTE to zero while increasing stiffness.

The primary disadvantage for the use of composites in mirrors is the highly irregular surface due to the presence of the stiffening fiber, known as "fiber print through." Also, in a honeycomb core construction, print through of the core cells can occur (as in lightweighted glass). Another problem is that large mirrors require butting multiple strips of composite material against each other. These butted joints can show up as bond lines and create undesirable features on the optical surface. From a structural point of view, composites are ideal for a mirror application. However, unless the issues of fiber print through and bond lines are solved, composites can not be used directly as an optical surface and may be limited in use as reaction structures for the mirror system. Possible solutions include thick polymer layers as the outer layer of the composite, claddings, carbon nanotubes, carbon nano fibers, and nanocomposites.

Composites that rely on continuous fibers for stiffness display anisotropic properties. Possible solutions for this problem include randomly oriented chopped fiber composites, nanocomposites, foam reinforced composites, and active shape control.

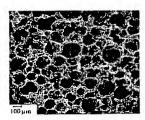
Borosilicate glass is a mass produced glass with a relatively low working point (~1200°C). Borosilicate glass is used extensively in small telescope applications because of its availability and low cost. However, its CTE is higher than desired for large mirror applications and its stiffness, like other glasses, is low. Borosilicate glass may be a good choice as a matrix material for a glass matrix composite with increased stiffness and tailored zero CTE. The high temperature required for production of a borosilicate matrix composite coupled with CTE mismatches may prevent the production of a net shape replicated mirror. Alternate glasses may be produced and processed into a composite. These include sol-gel derived glasses, plasma sprayed glasses, and water sprayable glasses such as potassium silicate.

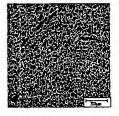
Since composites are typically comprised of polymer matrices and carbon fibers, like carbon and polymers, protective coatings are required to protect the composite from the low earth space environment.

Foam:

The optimal solution for producing web stiffened lightweighted structures is to produce many very small pockets that allow the thin facesheet to be supported without sagging. Machine tools and techniques limit how small these features can be made (~ millimeter level) and machining can not produce a fully optimized design. An open cell foam structure can provide pocket and structural member sizes at the micron level or smaller. Replacing a web structure with a foam for support of a mirror facesheet should yield a lighter, stiffer structure with no print through or quilting. Foams could be used strictly as a support for a bonded or deposited facesheet or as a reinforcement for a composite. Foams are available in silicon, silicon carbide, carbon, and other materials.

Production of a foam mirror could be accomplished by several methods. If the foam material can be foamed in place, a mirror could be produced by casting a thin film polymer or sol-gel glass / ceramic onto a convex paraboloidal mold and the foam formed directly behind it to rigidize the structure. If the foam requires an extensive or high temperature process, a mirror can be produced from a foam blank by machining, followed by the application of the optical surface to the foam by a sol-gel, melt, bonding, or deposition process. A composite designed with a foam could be designed to start as a bulk material for ease of replication or polishing, grade to a foam reinforced composite with low CTE and then grade to a foam with no matrix. This would optimize the weight reduction while ensuring no delamination of the facesheet can occur.





a) 200 ppi Graphitic Foam (0.15 g/cm³)

b) 500 ppi Graphitic Foam (0.8 g/cm³)

Figure 9. Microphotographs of Graphitic Foams [8]

Graphitic (carbon) foams (Figure 9.) may have utility in this application as a reinforcement for glass, ceramic, carbon, or polymeric surfaces. Research is ongoing to attempt to produce high stiffness, very low density carbon foams. However to this date, high stiffness carbon foam has not been demonstrated. Additional processing work is needed to determine if carbon foam can be produced with fully aligned graphitic planes to yield a lightweight and stiff product. Other possibilities for carbon foam include incorporation of high stiffness carbon foams as a stiffening agent to the foam. This would have the effect of producing a porous, stiff, lightweight composite for stiffening surface

materials. Additional work is required to determine if this is a viable concept.

Hybrid:

A near term solution for utilizing composites as mirror materials uses a hybrid approach. A hybrid approach allows for the combination of multiple non-optimal materials into a near optimal solution. An example of this is a lightweight, stiff polymer composite as a web stiffening structure or as a continuous paraboloidal structure can be used to support a very thin meniscus or coating of an optically ideal but stiffness poor material such as glass. Issues with this approach include: handlability of glass, CTE matching of facesheet with composite materials, bonding, and the complexity and cost due to two manufacturing processes for the mirror and the process for combining them.

Variable Emittance Coatings:

A technology that has the potential to eliminate high CTE as a concern for mirror materials is variable emittance coatings. CTE is an issue because of the shape change that occurs as a mirror changes temperature. The lower the CTE, the less the mirror is affected by temperature variations. A technology being pursued at The Air Force Research Laboratory is variable emittance coatings. The goal of ongoing programs is to produce a coating that can be electrically controlled to maintain a constant temperature by changing the thermal emittance of the external surfaces. These coatings, if fully developed, can be used on the back side of mirrors to control the mirror temperature to within a small range. This would allow the use of higher CTE materials such as polymers for use in membrane mirrors or Additionally, variable emittance segmented mirrors. coatings may allow temperature to be controlled over small sections of the mirror surface, allowing the CTE of the substrate to act as an actuator and control the mirror figure.

Mirror Coating Materials:

Mirror coatings have their own set of issues. Coatings must be applicable to the mirror facesheet material and supply both optical properties for the mirror and atomic oxygen protection for the substrate. Coatings can include metals (aluminum, silver, gold) for broadband reflectance, dielectric stacks or rugate coatings for wavelength specific (laser) high reflectance, or photonic band gap structures for very high, wide band reflectance. To supply protection from the low earth orbit space environment, coatings must be highly adherent, pinhole free, and themselves resistant to the space environment.

The major issue for coatings for large mirrors is how to apply them at a large scale. Segmented mirrors limit the size needed to coat, but the chambers must still be large to

accommodate the need of the whole segment (5m diameter). This means, vacuum chambers for applying these coatings may need to be larger than 5 meters in diameter.

Some coatings require a high temperature (~250°C) for application. This can lead to coating stresses, which can distort the mirror facesheet and can prevent lower temperature materials from consideration as facesheets. Alternate application methods that allow lower temperature deposition and stress free coatings may be required. Or, if a replication process is being used to produce the mirror, a major change in coating application technique may be possible. Since the release agents for replication may need to be applied by deposition techniques to the replication master, optical coatings could be applied onto the release agent rather than the final mirror. This would allow the reduction of the number of coating cycles and allow the application of coatings on previously uncoatable substrates (such as low temperature polymers). The coating would be transferred directly to the substrate material during the replication process. Multi layer coatings must be applied in the inverse order required to allow the proper function on the finished mirror.

Actuator Materials:

For segmented mirrors, position actuators are required to ensure alignment of the segments into a complete paraboloidal mirror. Typically, three actuators are required per segment to align the mirrors in tip, tilt, and piston. Additionally, as a segment becomes thinner due to lightweighting, additional actuators are required to adjust and maintain the shape of the individual segments. The less stiff the mirror facesheet, the more actuators are required to maintain the shape of the optical surface. Proper design of a mirror employs just enough actuators to bring mirror into compliance with mirror finish and radius of curvature requirements, without adding the additional weight of additional actuators. Lower mass actuators would allow the use of a greater number of actuators without a mass penalty. This would allow greater authority and control of the mirror figure as well as a higher first frequency vibration mode.

Current actuators are mechanical devices comprised of stepper motors, gears, and screws or electrostrictive or magnetostrictive materials. Figure 6 shows some common actuators. These actuators work by moving the mirror or applying a force to bend the mirror. Some problems with current actuator designs are high power requirements to drive the actuators, difficulty in control of the degree and repeatability of the force or position of the actuator, weight, and complexity.

Typical electrostrictive or magnetostrictive actuators consist of electrostrictive / ferroelectric materials such as lead zirconate-titanate ceramics (PZT), lead magnesium niobate (PMN), lead magnesium niobate-lead titanate (PMNPT),

and lanthanum modified lead zirconate-titanate; piezoceramics such as barium titanate (BaTiO₃) doped with iron; shape memory alloys such as Nitinol (NiTi) and palladium doped Nitinol (NiTi-Pd); and heterostructure multilayers of ferroelastic NiTi coupled to thin film SiO2 and ferroelectric PZT.

An alternative to current actuator designs is actuators based on Micro-electro-mechanical systems (MEMS) technology. Because of their very small size, MEMS actuators would allow a much higher density of actuation on a mirror facesheet which would allow greater control of the segment shape and the use of very thin, non-stiff facesheets such as polymer films which would allow production of very lightweight mirror segments.

Traditional actuation works perpendicular to the mirror surface. Actuators remove the large deformations of the mirror but instill multiple smaller deformations in the process. An actuation method that may allow better control of the surface is to apply force parallel or in a combination both parallel and perpendicular to the mirror surface. This would allow tightening of parts of the mirror that are sagging and loosening parts of the mirror that are too flat. This type of actuation would require quite a bit of software control but allow very precise shape control of the mirror. In order to supply this type of actuation, mechanical devices The actuation must be built into the can not be used. facesheet structure itself. One method to accomplish this is to incorporate shape change materials into the facesheet as a composite additive, bulk structural material, or coatings. The shape change material could be activated by electric or magnetic fields supplied by electrical grids, electron guns, or other methods. In addition to the electrostrictive or magnetostrictive materials used for conventional actuators, materials that could function in this application include electroactive polymers such as conductive polymers, ionic polymer gels and liquid crystal polymers. This materials approach to actuation would greatly reduce the mechanical complexity, cost, and weight of actuators. This approach is critical for shape management of thin film membrane mirrors, since traditional mechanical actuation would not be possible.

Reaction structure:

Currently, a stiff reaction structure is required to hold the actuators, which, in turn, hold the mirror facesheet. Much of the weight in a mirror system is found in the reaction structure (typical mirror mass budget: 1/3 for facesheet, 1/3 for actuators, and 1/3 for reaction structure). Typical materials for reaction structures include graphite / cyanate ester composites and beryllium. If actuators can be built into the mirror facesheet that react parallel to the mirror surface rather than perpendicular, the reaction structure only needs to deploy and hold the segmented or membrane mirror. It would be possible to replace the stiff reaction structure with

an inflatable, rigidizable reaction structure. The advantage of this is further weight savings, simplified deployment methods, and elimination of mechanical joints and devices such as hinges, springs, and motors. The inflatable structure could be designed to align any individual segments without the need for additional position actuators.

3. CONCLUSIONS AND RECOMMENDATIONS

The use of conventional materials such as glass to produce large, lightweight, mirrors leads to non-optimal designs, which require a long lead time and high cost to procure. New materials, processes, and manufacturing methods are required to meet AF needs.

There are many possible avenues to pursue for research and development of new materials, processes, and manufacturing methods for large space mirrors. However, if efforts that address multiple issues (such as low mass, high stiffness, and good manufacturability) are considered as higher priorities, a more manageable list of high priority research and development (R&D) areas can be produced.

Machining is the predominant cost and schedule issue for the procurement of current state of the art mirrors. Any R&D into mirror materials should consider materials that are easier to procure and machine. Any R&D into mirror manufacturing should address decreasing the cost and time of machining of mirror facesheets. If possible, multiple machining steps needs to be eliminated or reduced in order to reduce cost. This leads to the need for materials that can be net or near net shape formed as much as possible. Techniques that lead to this include replication methods and spin casting methods. These areas would provide a large return on investment if R&D dollars were invested..

For rigid, segmented mirrors, high specific stiffness is critical for achieving lightweight, dimensionally stable mirrors. Materials that meet this criterion include composite or hybrid materials stiffened with high modulus graphite fibers. A wide variety of matrix materials are available to produce composite materials. The choice should be made to investigate only those matrix materials that lend themselves to a replication or spin casting process. These include polymers, low temperature glasses, and sol gel derived ceramics.

For membrane mirrors, it is critical that the membrane be capable of being stowed in a small deployable package. Therefore, the membrane material must be sufficiently flexible to allow rolling or folding but strong enough to resist stretching or deforming when deployed. Materials to consider for this use include many polymer materials and possibly thin metal films, as well as possibly metal films reinforced with polymer layers.

Since both rigid and membrane mirrors can potentially make use of polymers as an optical substrate, efforts should be initiated to address the apparent show stoppers for using polymers for these applications. These include high CTE, low stiffness, fiber print through of polymer matrix composites, and moisture loss and outgassing in vacuum. In addition to developing a low CTE polymer, an alternate approach is to use conventional polymers and control mirror temperature by the use of variable emittance coatings. Efforts to further develop this material concept should continue.

Finally, coating systems to allow the adaptive shape control and/or deployment of membrane optics are required. Efforts are required to develop adaptive materials and fashion them into multilayer, controllable coating systems.

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5. BIOGRAPHY

Pat Carlin is a materials engineer for the Air Force Research Laboratory's (AFRL) Materials and Manufacturing Directorate (ML). Pat's technical background includes over 12 years experience as a program manager and focal point leader in the area of tribology and coatings. His main area of



expertise has been thermal control coatings for spacecraft. As program manager, Pat was responsible for the definition and validation of requirements, defining appropriate in-house research projects as well as contracted efforts, and executing the technical and business strategies for contracted efforts. Pat was also responsible for the redesign and upgrade to the Space Combined Effects Primary Test and Research Equipment (SCEPTRE) Facility at the Materials and Manufacturing Directorate. The facility is designed to test

the space stability of thermal control materials. Pat currently serves as a colocated engineer to the AFRL Philips Research Site, where he provides the main interaction between ML and our space and directed energy customers and partners at Phillips Research Site. Pat's responsibilities include providing materials engineering support, representation, coordination, and leadership ensuring that leading edge materials and processes technology is provided to the Air Force's space and directed energy customers. Pat is responsible for defining ML program requirements and recommending investment strategy for space and directed energy weapons. This includes: reviewing and assessing materials needs and requirements across a broad range of space and directed energy customers, reviewing and assessing industry and other government lab technology development, and assessing opportunities or requirements in new materials technology areas.

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